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SEMINAR

Grupo de Análise Funcional e Aplicações
Functional Analysis and Applications Group

Convolution equations and BVPs for the Generic Laplacian on Lie groups

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Abstract

Lie group $\boldsymbol{G}$ is a manifold where each element has the inverse $x \circ x^{-1} = e$ and e is the neutral element (identity). Then on $\boldsymbol{G}$ we have a unique invariant Haar Measure $d_{\boldsymbol{G}}x$, Fourier transformation $\mathcal{F}_{\boldsymbol{G}}$ with the inverse $\mathcal{F}_{\boldsymbol{G}}^{-1}$ and associated Generic differential operators $\mathfrak{D}_1, \dots, \mathfrak{D}_n$, generated by the vector fields from the associated Lie algebra. We consider commutative (Abelian) Lie groups, homeomorphic to $\mathbb{R}^n$ and, therefore, the dual group $\widehat{\boldsymbol{G}}$ coincides with $\mathbb{R}^n$. That allows definition of convolution integro-differential equations

$$(W^0_{\boldsymbol{G},a}\varphi)(x) := \mathcal{F}^{-1}_{\boldsymbol{G}} a \mathcal{F}_{\boldsymbol{G}}\varphi)(x) = \sum_{||\alpha|\leqslant m} \left[c_\alpha \mathfrak{D}^\alpha \varphi(x) + \int_{\boldsymbol{G}} k_\alpha(x \circ y^{-1}) \mathfrak{D}^\alpha \varphi(y) d_G y\right] = f(x), \quad x \in \boldsymbol{G}$$

with the symbols of polynomial growth

$$a(\xi) := \sum_{|\alpha|\leqslant m} \left[c_\alpha (-i\xi)^\alpha + (-i\xi)^\alpha \mathcal{F}_{\boldsymbol{G}} k_\alpha(\xi)\right], \qquad \xi \in \widehat{\boldsymbol{G}} = \mathbb{R}^n$$

and of the Generic Bessel Potential Spaces $\mathbb{GH}^s_p)(\boldsymbol{G}, d_{\boldsymbol{G}}x)$, endowed with the norm $\|\varphi\big|\mathbb{GH}^s_p)(\boldsymbol{G}, d_{\boldsymbol{G}}x)\| = \|\mathcal{F}^{-1}_{\boldsymbol{G}}\langle\xi\rangle^s \mathcal{F}_{\boldsymbol{G}}\mathcal{F}^{-1}_{\boldsymbol{G}}\big|\mathbb{L}_p(\boldsymbol{G}, d_{\boldsymbol{G}}x)\|$, $\langle\xi\rangle^s := (1+|\xi|^2)^{s/2}$. In this framework we study convolution integro-differential equation (1) in the setting

$$\varphi \in \mathbb{GH}^s_p(\boldsymbol{G}, d_{\boldsymbol{G}}x), \qquad f \in \mathbb{GH}^{s-m}_p(\boldsymbol{G}, d_{\boldsymbol{G}}x) \tag{1}$$

and Boundary Value Problems (BVPs) on domains $\Omega \subset \boldsymbol{G}$ for the Generic Laplacian:

$$\Delta_{\boldsymbol{G}}(\mathfrak{D})\psi(x) = h(x), \quad \Delta_{\boldsymbol{G}} := \mathfrak{D}_1^2 + \ldots + \mathfrak{D}_n^2 \quad x \in \Omega, \quad \psi \in \mathbb{GH}^s_p(\boldsymbol{G}, d_{\boldsymbol{G}}x),$$
$$(P(\mathfrak{D})\psi)^+(t) = g(t), \quad t \in \Gamma := \partial\Omega, \quad h \in \mathbb{GH}^{s-2}_p(\boldsymbol{G}, d_{\boldsymbol{G}}x), \quad g \in \mathbb{GH}^{s-r}_p(\boldsymbol{G}, d_{\boldsymbol{G}}x)$$

where $P(\mathfrak{D})\varphi = \varphi$ (the Dirichlet problem) or $P(\mathfrak{D})\varphi = \partial_\nu\varphi$ (the Neumann problem) and ν is the unit normal vector field on the boundary Γ. As an example we consider the Lie group $I = (-1, 1)$, where the group operation is $x \circ y = \dfrac{x+y}{1+xy}$, neutral element is 0, the inverse to $x \in I$ is $-x$, the Haar measure is $d_I x = \dfrac{dx}{1-x^2}$ and the generic differential operator is $\mathfrak{D} := (1-x^2)\dfrac{d}{dx}$.

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